ON THE TRANSMISSION OF WALSH MODULATED MULTIPLEX SIGNALS

H. Hübner

Research Institute

of the

Telecommunication Engineering Centre

of the Deutsche Bundespost

Darmstadt, Western Germany

Introduction

The carrier frequency systems cu-
stomary in communication engineering
transmit information in the frequency
multiplex and use circular functions
as carriers. The respective underlying
principle is also transferable to the
system of orthogonal Walsh functions.
The theory of the signal multiplexing
technique based on the use of Walsh
carriers as functions is dealt with
in [1]. A number of possible set-ups
of multiplex systems and various modes
of operation are described in [2].

By way of theoretical deliberations
it was shown that a sequency system is
capable of transmitting sequency-limi-
ted signals without any distortion
provided the system is composed of
ideal components and provided a trans-
mission channel is used which is mat-
ched to the sequency multiplex signal.
Moreover, also signals of limited
frequency bandwidth can correctly be
transmitted with such a system if
appropriate matching circuits inserted
before the sequency multiplex equip-
ment transform the information into
pseudo sequency-limited signals in
consideration of the sampling theorem
and if other additional circuits re-
transform the signals to their origi-
nal shape after the transmission.

The set-up of a standard multiplex
equipment is shown in Fig. 1. Its
functioning can be seen from the course
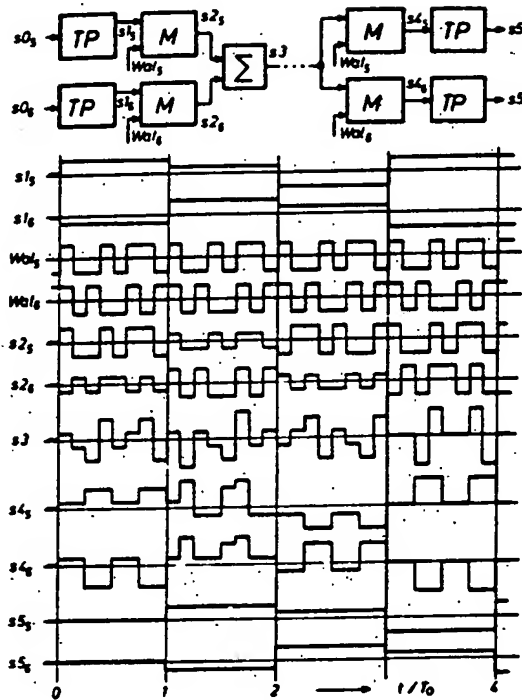of the signal diagrams at different
stages in the system.

Fig. 1: Principle of a sequency
multiplex system
TP sequency low-pass filter,
M multiplier

Distortion of Signals and Crosstalk

In contrast ot the conditions pre-
vailing in the case of a multiplex
transmission under ideal circumstances,
signal distortion and crosstalk will
generally appear whenever real system
components are used. Even if the trans-
mitter and the receiver are made up of
distortion-free ideal components there
will be some distortion due to the fact
that the bandwidth of customary trans-
mission media is always finite and due

to differences in the synchronism bet-
ween transmitter and receiver. Both
cause distortions of the signal trans-
mitted as well as crosstalk on adjacent
channels.

The subsequent investigations are
based on the assumption of linear trans-
mission channels with low-pass charac-
teristic. Here, Gaussian and rectan-
gular-shaped transmission factors with
linear phase response are taken as
borderline cases of possible frequency
limitation. They approximate extreme
real conditions. Although these assump-
tions do not lead to causal systems
they yield experience which can suitab-
ly be employed for assessing the pro-
perties of realistic equipment. The
test signal employed for the deter-
mination of the system behaviour is a
pulse whose duration equals the time
base T_0 as used in the multiplex
system. It is the "quantum" of a signal
of limited sequency bandwidth and it
permits the formation of any series of
signals. The system behaviour opposite
other input signals can, therefore, be
derived from the respective superposi-
tion of the results obtained for the
test signal since the multiplex system
and the channel are both linear arran-
gements.

Standard Multiplex System

Fig. 2 shows the set-up of a simple
sequency multiplex equipment. It is
similar to the arrangement presented
in Fig. 1 and it also evaluates the
transmitted multiplex signal in the
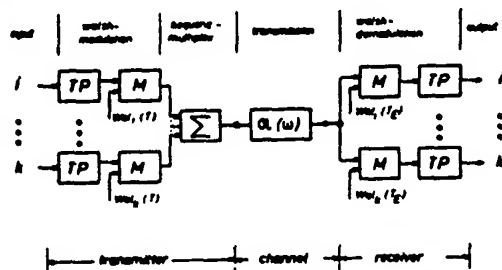receiver during the demodulation
process by integration over T_0.

Fig. 2: Principle of a standard
multiplex system

The distortions occuring in a
4-channel system with accurate syn-
chronization between transmitter and
receiver are plotted in the Figs. 3

and 4. Fig. 3 shows the amplitude of
signal sa_{ii} at the output i of the
syst m which is produced by the test
signal se at the input i of the sy-
st m in dependence of the cut-off
frequency F_g of the transmission
channel. In this case, a rectangular
pass band was assumed. Fig. 4 illus-
trates the signal distortions for a
channel with a Gaussian type of trans-
mission factor. The Figs. 5 and 6
show the maximum crosstalk from chan-
nel i to channel j as produced by se
due to frequency limitation in the
transmission path in the case of a
rectangular and Gaussian transmission
factor. In all four diagrams $F_g = T_0 \cdot f_g$
is a parameter normalized to the time
base T_0. If the time base is assumed
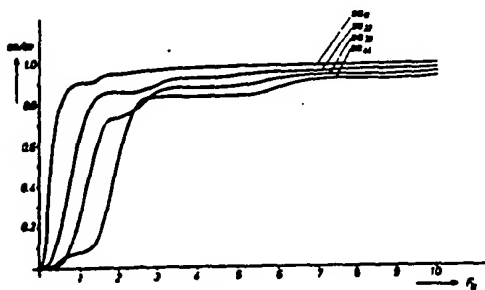to be 125 /us then the expression
reads: $f_g = 8 \cdot F_g$ kHz.

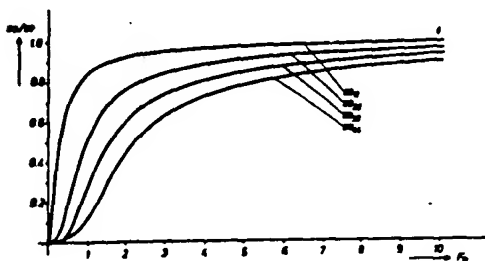Fig. 3: Output sa_{ii} caused by a
rectangular shaped channel

Fig. 4: Output sa_{ii} caused by a
Gaussian shaped channel

The diagrams indicate that with
rectangular limitation of the fre-
quency band less signal distortion
and more crosstalk is obtained than
with a band limitation of the Gaussian
type. For low-distortion transmission,
the standard sequency multiplex equip-
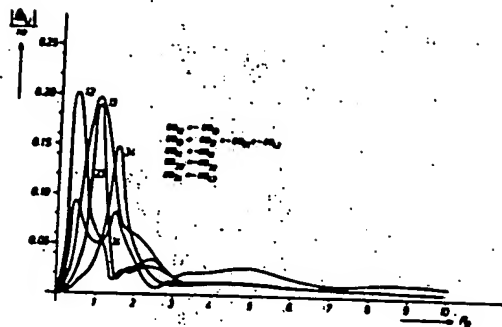ment requires a channel of very large
frequency bandwidth.

42

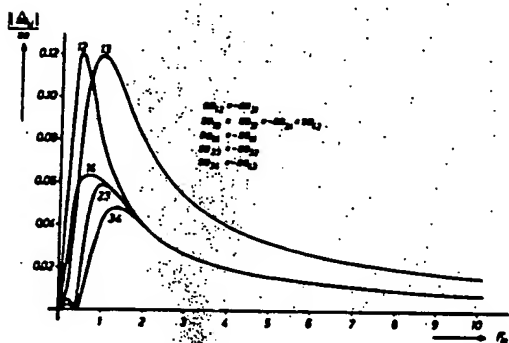Fig. 5: Crosstalk caused by a
rectangular shaped channel

Fig. 6: Crosstalk caused by a Gaussian
shaped channel

The results obtained refer to a
multiplex equipment of accurate syn-
chronization. There, the frame clock
controlling the sequency low-pass
filter and the interval of orthogo-
nality of the Walsh carriers both in
the transmitter and in the receiver
are not delayed with respect to time.
Moreover, the frame clocks of trans-
mitter and receiver are synchronous
and the time delay of the signals was
taken into account accordingly in the
calculations. In the case of real
systems this ideal condition generally
is not given. By an appropriate set-up
of th transmitting and receiving
equipment it can, however, be achieved
that the frame clock and the pattern
of th Walsh carriers coincide with
sufficient accuracy. The exact phase-
locked coupling of the synchroniza-
tion between transmitter and receiver,
on the other hand, is only possible
within certain limits. The investi-
gation of the effects of this type
of synchronization errors on the
system response shows that it also
caus s signal distortions and cross-
talk. Their extent depends on the

limitation of the fr qu ncy bandwidth
of spectrum of the multiplex signal
and differs on the individual chann ls.

Transmission rr rs n be reduced
if the multiplex signal transmitted is
sampled at appropriate times and if it
is restored to its original steplike
shape prior to the demodulation in the
receiver. This can be achieved by means
of a sampling and holding circuit pro-
vided at the input of the receiver.
The study of this extended type of
multiplex equipment indicates a clear
improvement of the system behaviour as
against the standard multiplex equip-
ment. For a low-distortion transmission
the extended system does, however,
always require a channel of large fre-
quency bandwidth and remains sensitive
to synchronization errors.

Sequency Multiplex System With PAM

The above mentioned investigations
showed that a direct transmission of
Walsh modulated signals via customary
frequency limiting media leads to
errors which cannot be neglected. The
origin of these errors lies in the
shape of the multiplex signal. The
system response can be improved if
this signal is appropriately matched
to the characteristics of the channel.
One possibility of achieving this is
to subject the sequency multiplex sig-
nal to an additional pulse amplitude
modulation process before the trans-
mission. Fig. 7 shows such an extended
multiplex equipment. According to this
scheme, the Walsh multiplex signal
modulates the amplitude of the pulse
carrier $P(T)$. Following the trans-
mission of the PAM signal the receiver
restores the sequency multiplex signal
to its original shape by regenerating
it from the individual pulses with
the aid of a sampling and holding cir-
cuit. During the regeneration an
additional amplification of the pulse
amplitudes is required. It compensates
the reduction of the pulse amplitudes
which is inevitably obtained whenever
pulses pass through a low-pass filter
having the cut-off frequency F_g. In
the case of narrow pulses the amplifi-
cation v must be inversely proportio-
nal to F_g.

The distortions occurring in a
system of 4 channels with exact syn-
chronization between transmitter and
receiver are shown in Fig. 8 and Fig.9
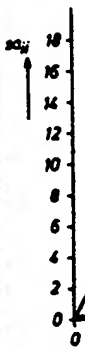for r ctangular and Gaussian channel

Fig. 7:

Fig. 8:

Fig. 9:

transm
calcul
select
vidual
sequenc
signal
ted si
plotte
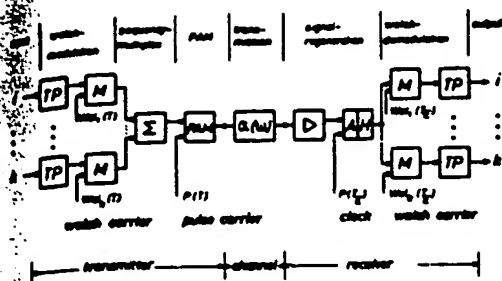measur
ver pr
value
system

Fig. 7: Principle of a sequency multiplex system with PAM
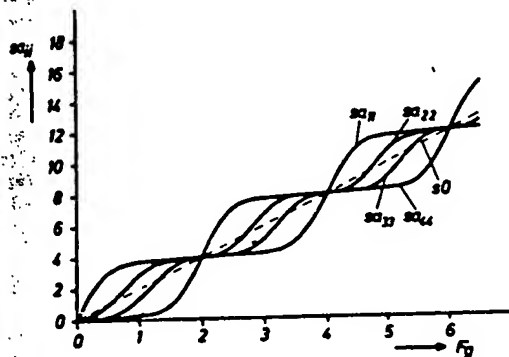

Fig. 8: Output sa_{ii} caused by a rectangular shaped channel
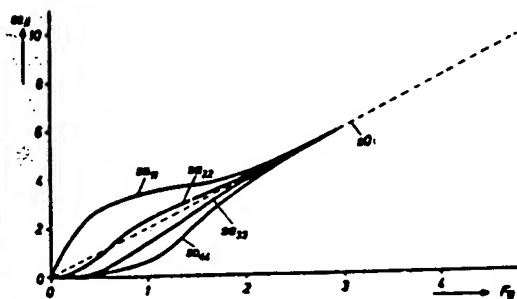

Fig. 9: Output sa_{ii} caused by a Gaussian shaped channel

transmission factors. For the relevant calculations a value of $T_0/256$ was selected for the width of the individual rectangualr pulses of the pulse sequence P(T). In addition to the signals $sa_{ii} = g(F_g)$ also an undistorted signal amplitude $s0 = g(F_g)$ is plotted in both Figures. It could be measured at the output of the receiver provided the amplification had the value 1. Fig. 8 illustrates that the system does not cause any distortion

of signals if the rectangular channel frequency bandwidth F_g is even-numbered. Even if the channel is assumed to have a transmission factor of the Gaussian type any amplitud distortions in the signals sa_{ii} will disappear very quickly as can be seen from Fig. 9.

The crosstalk values determined for the system are plotted in the Figs. 10 and 11. The diagrams indicate that the crosstalk disappears under the same frequency conditions where also the signal distortions in sa_{ii} become negligible.
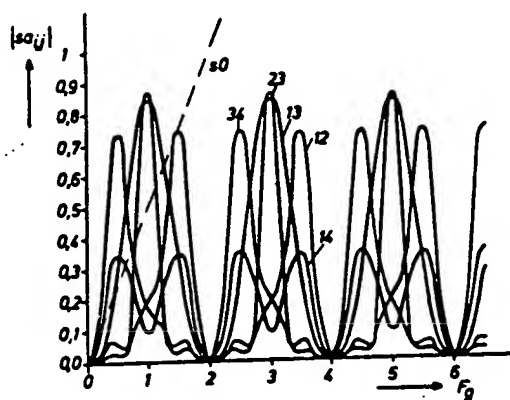

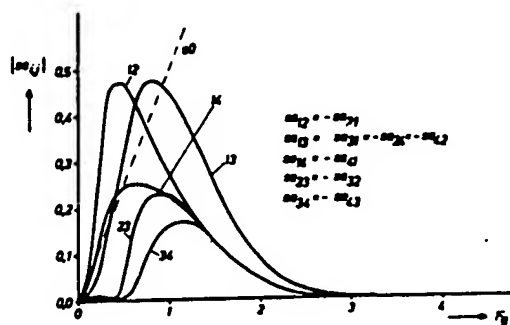Fig. 10: Crosstalk caused by a rectangular shaped channel

Fig. 11: Crosstalk caused by a Gaussian shaped channel

The favourable system characteristics offered by a transmission equipment according to Fig. 6 become all the more apparent if the thus normalized values for the signal distortions and for the crosstalk are compared to the respective parameters of the standard multiplex system.

44

The behaviour of the multiplex equipment deteriorates as soon as there are disturbances in the synchronism between transmitter and receiver. An example is given in Fig. 12 where the percentage of the signal distortion sa_{11} with the cut-off frequencies $F_g = 2$ and $F_g = 4$ as parameters is plotted over the time shift τ/T_0 in the synchronism. Further details concerning the behaviour of sequency multiplex equipment with additional pulse amplitude modulation can be taken from [3].
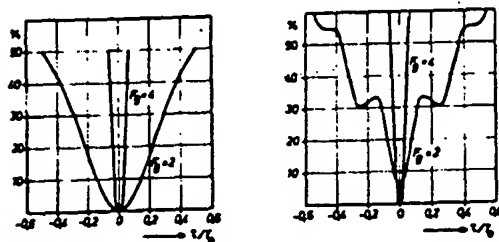
Fig. 12: Interference of the output sa_{11} by synchronization delay
left: rectangular shaped channel
right: Gaussian shaped channel

Summary

It was shown that a direct transmission of Walsh modulated signals via channels with low-pass characteristic generally leads to signal distortions and crosstalk. The amount of interference obtained during transmission strongly increases with decreasing frequency bandwidth. Disturbances in the synchronism between transmitter and receiver also result in signal distortion. The theoretically determined system behaviour was actually confirmed by measurements made with a sequency multiplex equipment.

An improvement of the transmission quality is possible by matching the multiplex signal to the channel characteristics. This can, for example, be achieved by means of an additional pulse amplitude modulation process. The investigation of such an extended sequency multiplex system shows that the transmission errors disappear if exact synchronism is ensured and if an appropriate channel bandwidth is selected.

References

[1] Harmuth, H.F.: Transmission of Information by Orthogonal Functions.
Springer Verlag Berlin, Heidelberg, New York, 1969

[2] Hübner, H.: Analog und Digital Multiplexing by Means of Walsh Functions
Symposium and Workshop, Naval Research Laboratory and University of Maryland, March 31. - April 3. 1970, Washington D.C.

[3] Lovis, W.: Berechnung der Ausgangssignale an einer Sequenz-Multiplexeinrichtung mit zusätzlicher Puls-Amplituden-Modulation
Thesis submitted to the Institute for General Communications, Technische Hochschule, Darmstadt, Western Germany, December 8. 1969.